Advanced Electronic Communication Systems

Lecture 3

Satellite Orbits (Part 2)

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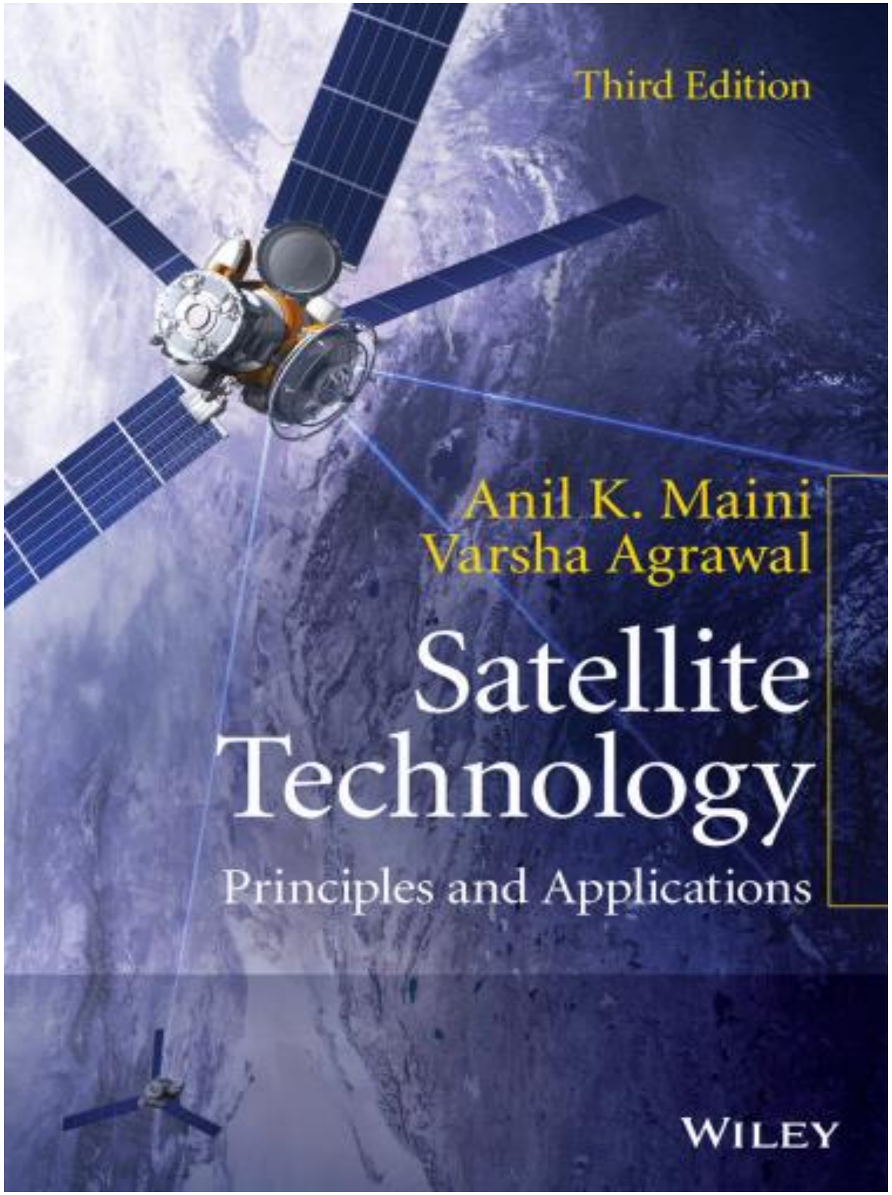


Chapter (2)
Satellite Technology: Principles and Applications

Kepler's Third law (Harmonic law or law of periods)

The square of the time period of any satellite is proportional to the cube of the semi-major axis (α) of its elliptical orbit of its orbit.

- The expression for the circular time period can be derived by equating the gravitational force with the centrifugal force:

$$\frac{Gm_1m_2}{r^2} = \frac{m_2v^2}{r} \quad (2.14)$$

Replacing v by ωr in the above equation gives

$$\frac{Gm_1m_2}{r^2} = \frac{m_2\omega^2r^2}{r} = m_2\omega^2r \quad (2.15)$$

which gives $\omega^2 = Gm_1/r^3$. Substituting $\omega = 2\pi/T$ gives

$$T^2 = \left(\frac{4\pi^2}{Gm_1}\right)r^3 \quad (2.16)$$

This can also be written as

$$T = \left(\frac{2\pi}{\sqrt{\mu}}\right)r^{3/2} \quad (2.17)$$

Kepler's Third law

- This equation holds for elliptical orbits by replacing r with α

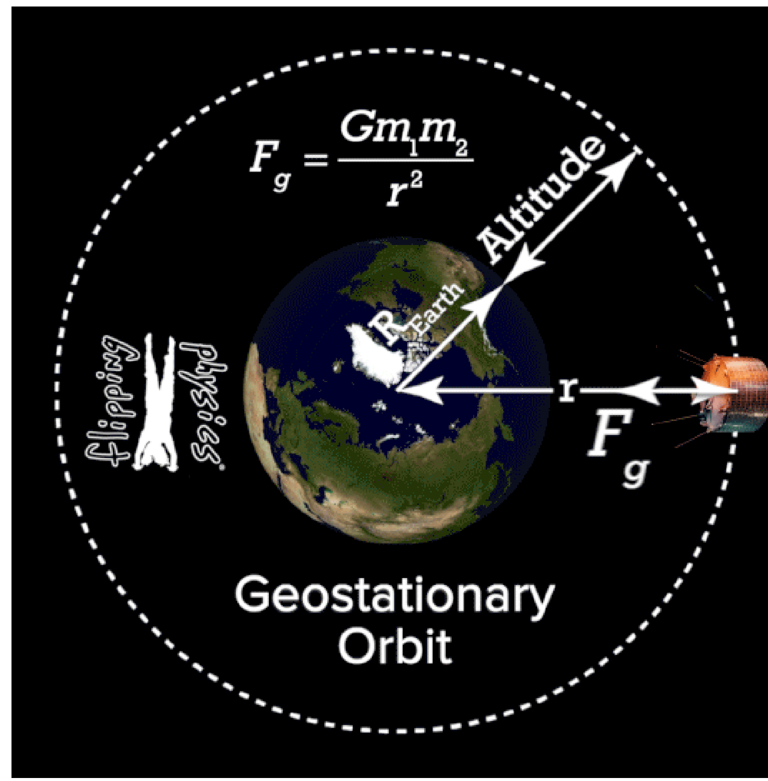
$$T = \left(\frac{2\pi}{\sqrt{\mu}}\right)\alpha^{3/2}$$

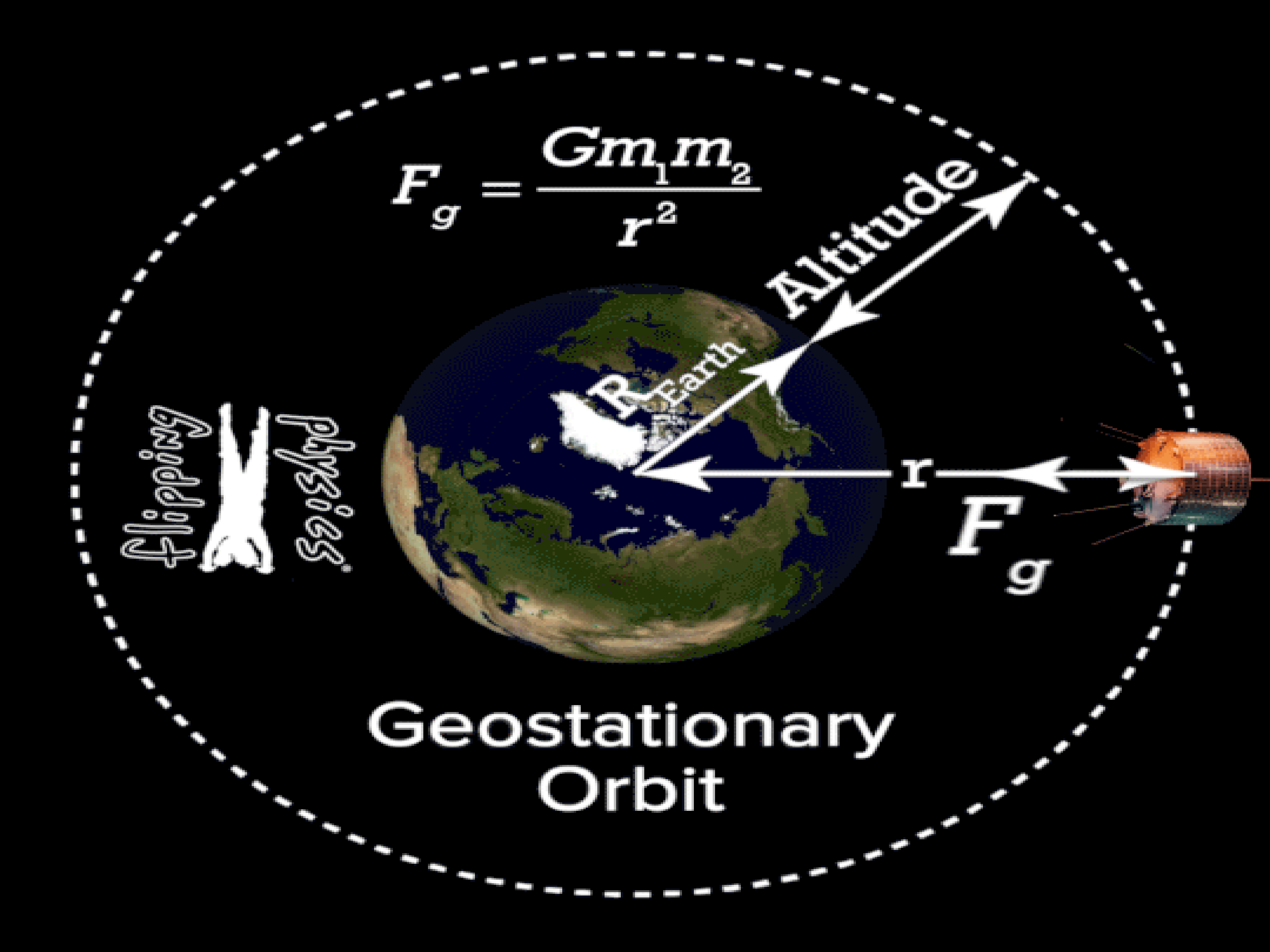
- It can be written in terms of ω (angular velocity in rad/sec)

$$\alpha^3 = \frac{\mu}{\omega^2} = \frac{\mu}{n^2}$$

- Some references uses symbol "n" instead of "ω"

- This law allows the satellite designer to select orbit periods, which best meet particular application requirements by locating the satellite at the proper orbit altitude.
- One very important orbit in particular, known as the geostationary orbit (42,241 Km), is determined by the rotational period of the earth (almost 1 day)





$F_g = \frac{Gm_1m_2}{r^2}$
Altitude
R_{Earth}
r
F_g
Geostationary Orbit

Kepler's Third law

Example 2.1 Calculate the radius of a circular orbit for which the period is 1 day.

Solution There are 86,400 seconds in 1 day, and therefore the mean motion is

$$n = \frac{2\pi}{86400}$$

$$= 7.272 \times 10^{-5} \text{ rad/s}$$

From Kepler's third law:

$$a = \left[\frac{3.986005 \times 10^{14}}{(7.272 \times 10^{-5})^2}\right]^{1/3}$$

$$= \underline{\underline{42{,}241 \text{ km}}}$$

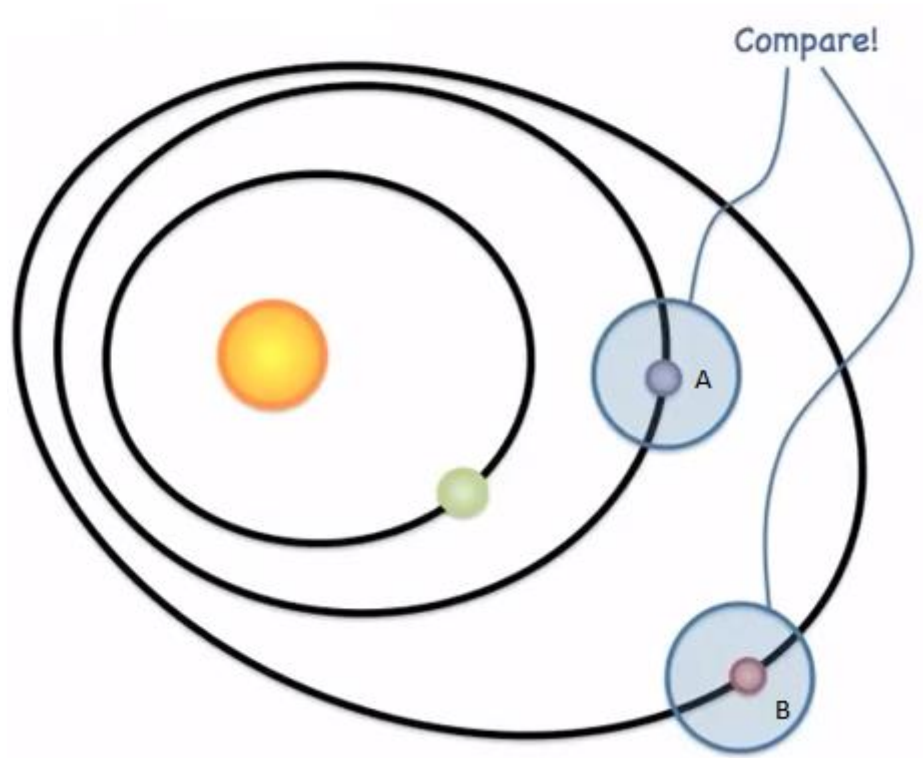
Since the orbit is circular the semimajor axis is also the radius.

- Actually, it takes not 24h exactly but 1 sidereal day (23h, 56min, 4 sec),
- The actual distance is 42164 Km

Kepler's Third law

➢ Comparison between multiple orbiting objects around the same body:

$$(T_A/T_B)^2 = (\alpha_A/\alpha_B)^3$$



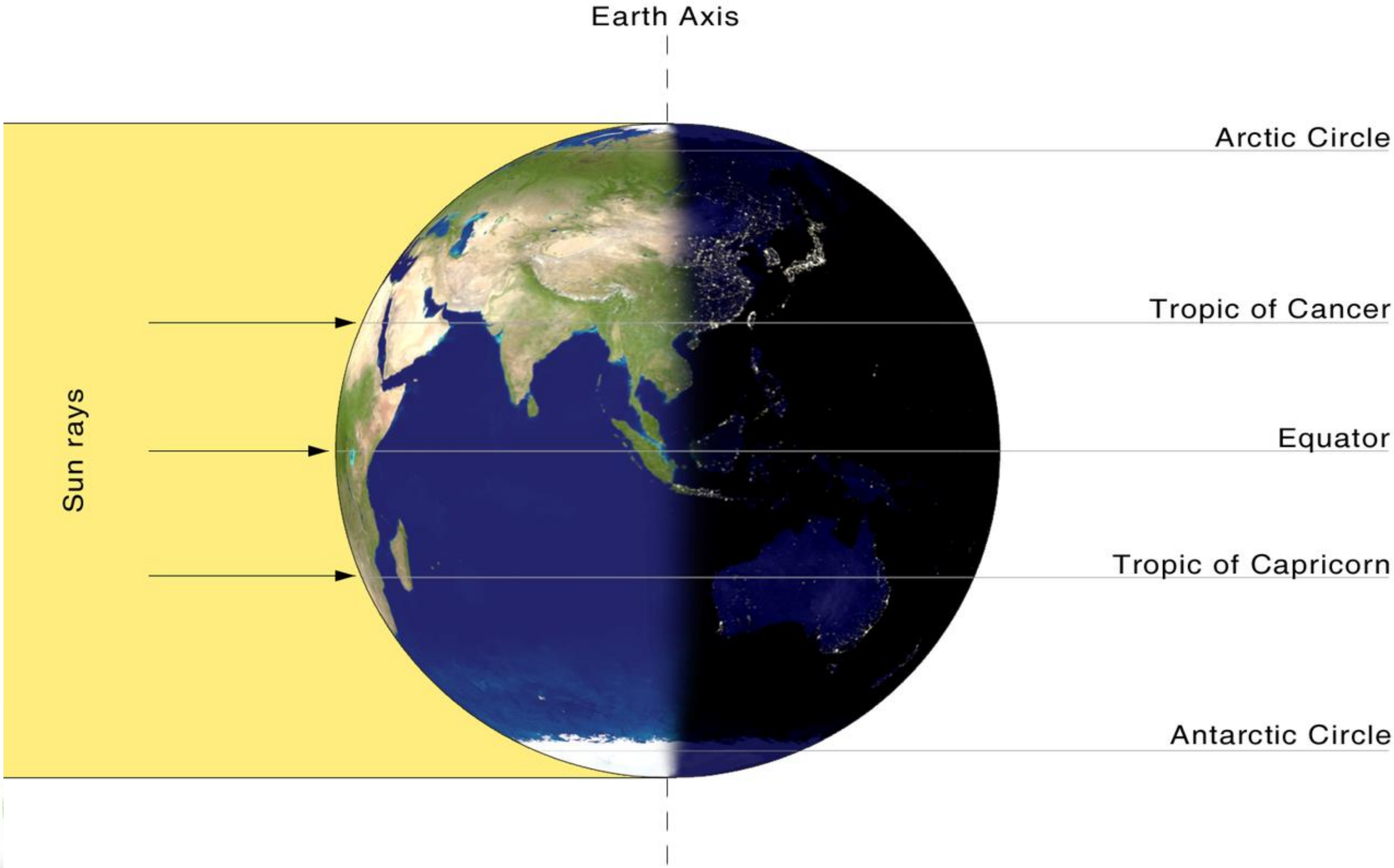
Orbital Parameters

- There are several parameters the define the orbit

1. Ascending and descending nodes
2. Equinoxes
3. Solstices
4. Apogee
5. Perigee
6. Eccentricity
7. Semi-major axis
8. Right ascension of the ascending node
9. Inclination
10. Argument of the perigee
11. True anomaly of the satellite
12. Angles defining the direction of the satellite

Basic Definitions:

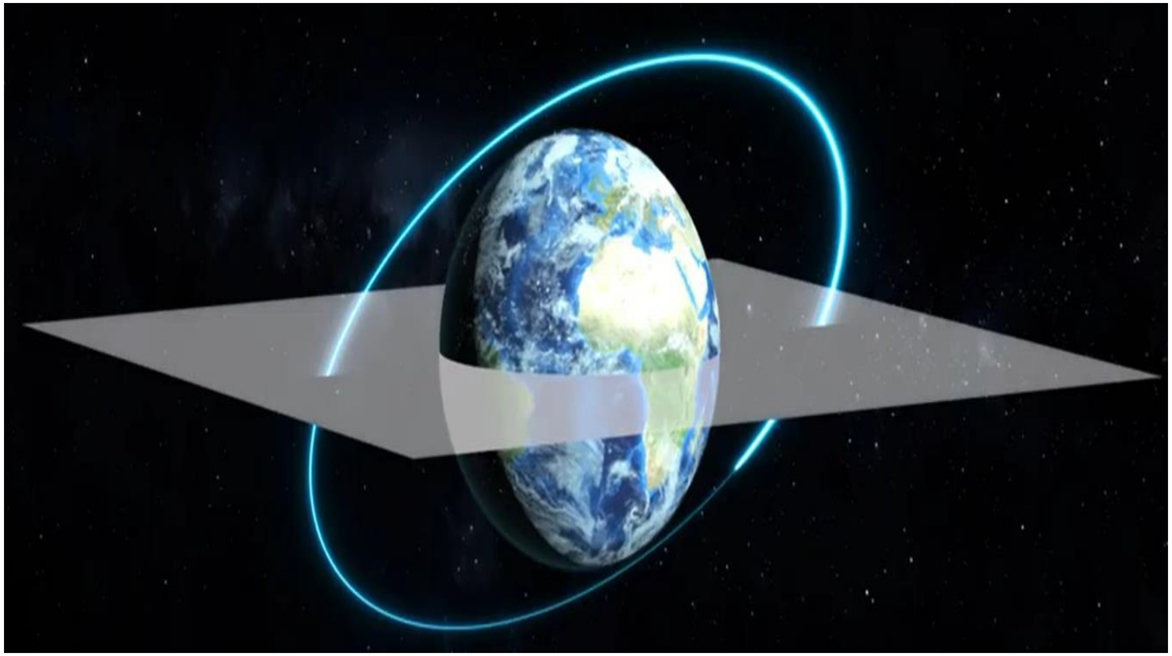
Earth Circles and Axis



Basic Definitions:

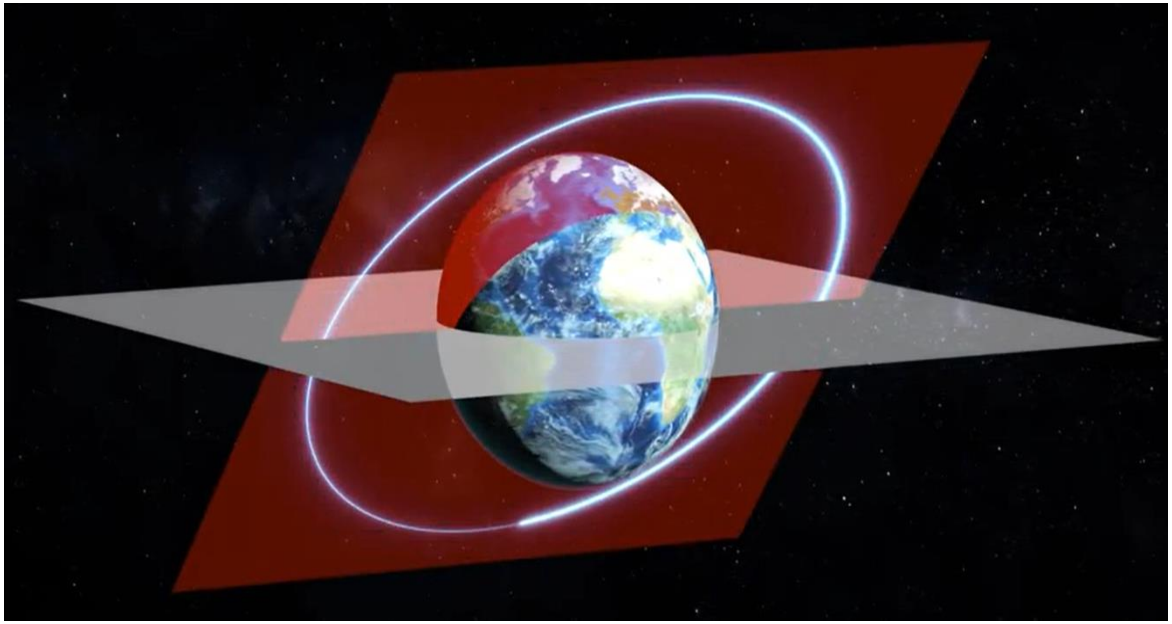
Earth Equatorial Plane

The plane that passes by the center of the earth and extends out through the equator



Satellite Orbital Plane

The plane that flat on top of the satellite orbit and passes through the center of the earth



Basic Definitions:

Satellite's orbit always has to intersect the equator, why?

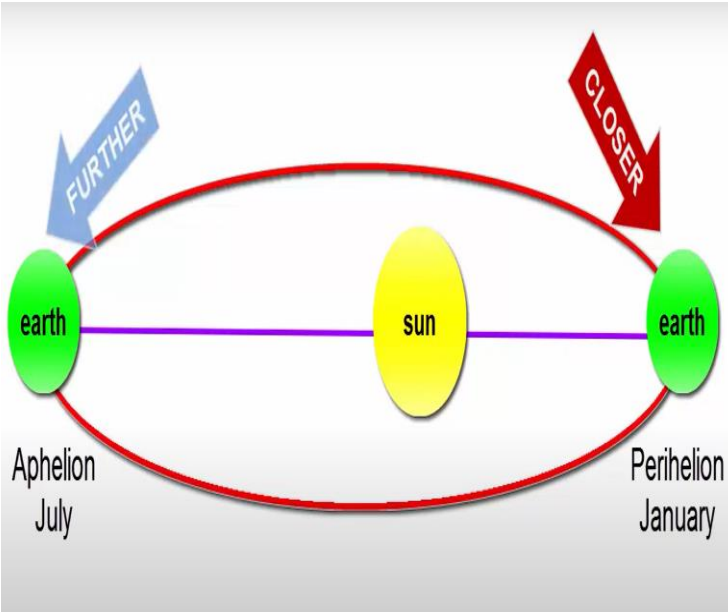
There's a very simple reason:

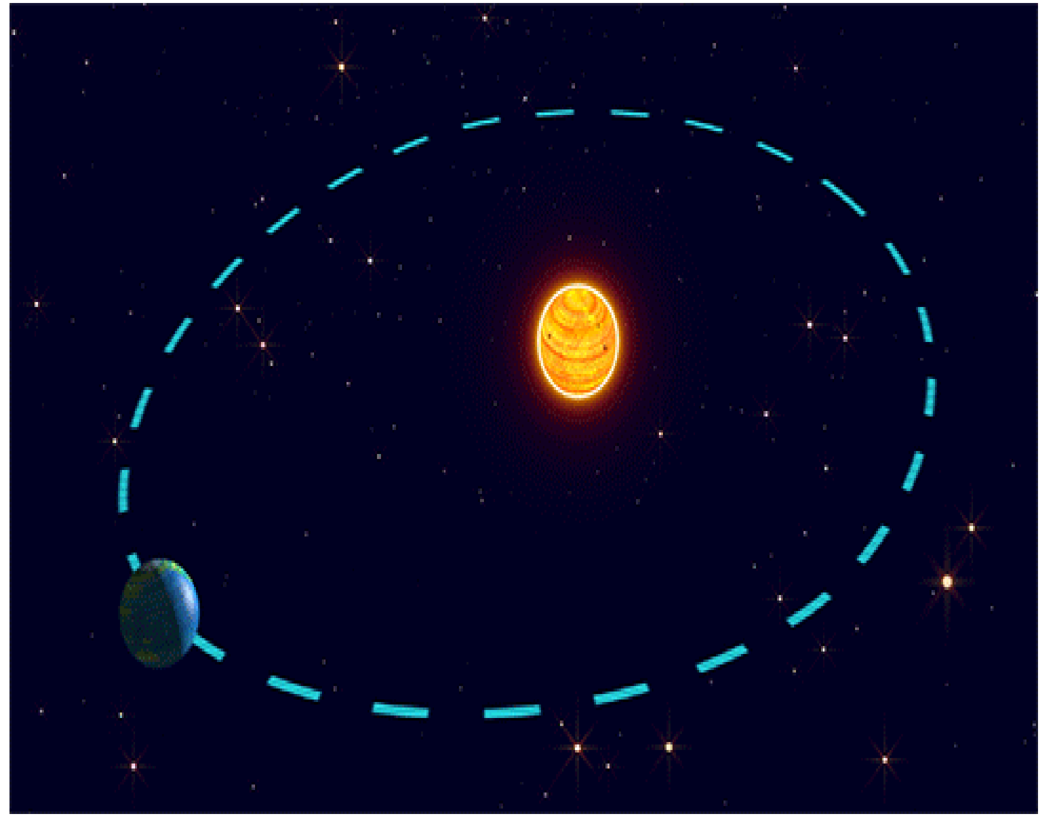
- The plane of the orbit must always contain the center of the Earth, which is a part of the earth's equatorial plane, **why again?**
- The answer is in Kepler's First Law :

The center of gravity of the celestial body around which the satellite is in orbit must always be at one focus of the ellipse that is the orbit

Basic Definitions:

Earth Motion

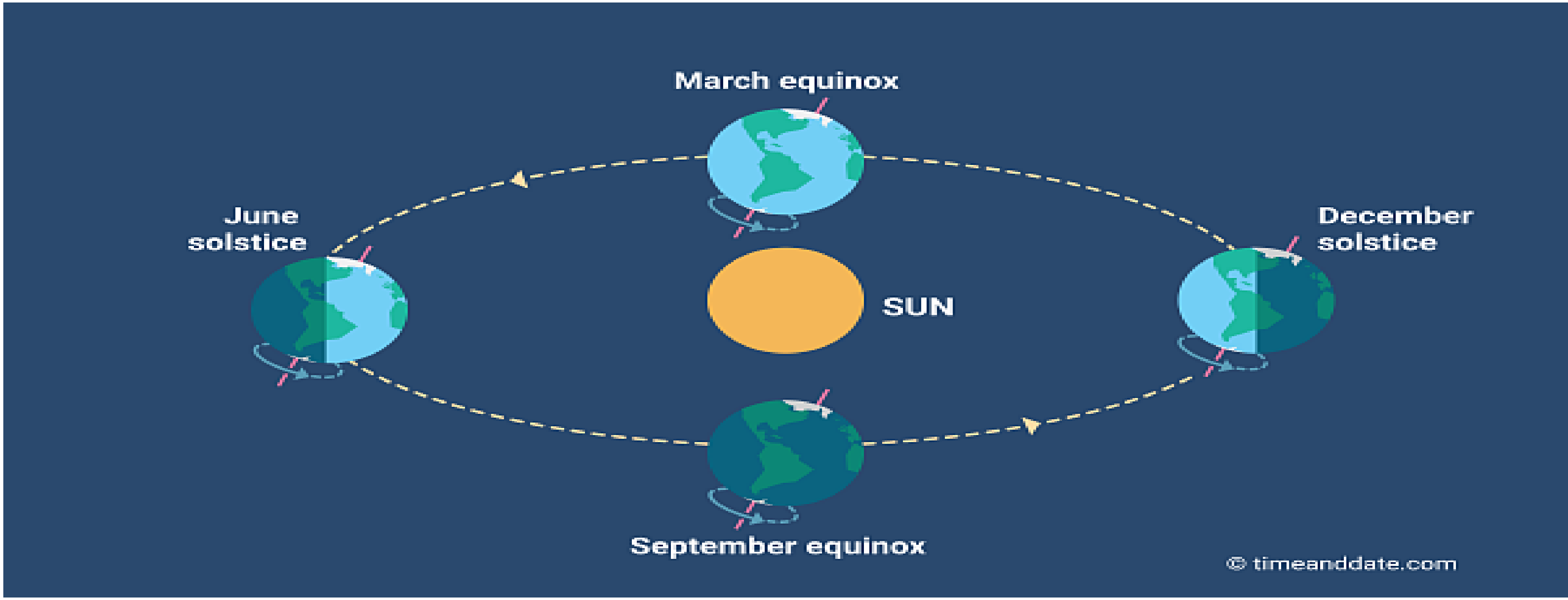




- Rotation: Earth rotates about its axis daily

- Revolution: Earth revolves around the sun in elliptical orbit in 365.25 days, where the nearest point is 147 million Km and the furthest point is 152 million Km.

Basic Definitions: Earth Titled Axis Effect

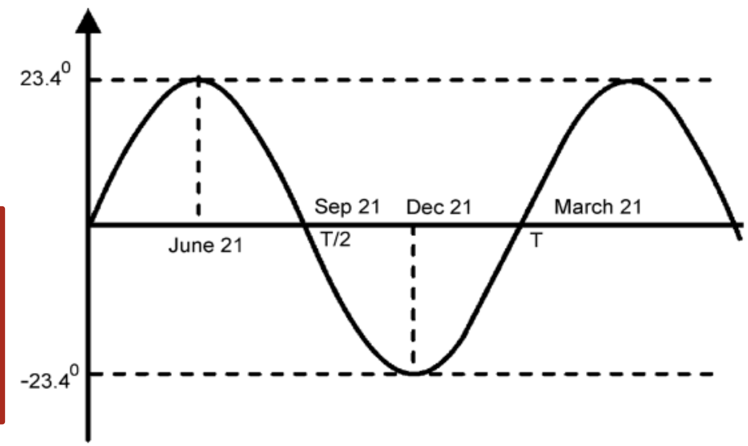


- Our planet normally orbits the sun on an almost fixed axis that's tilted 23.43^o, due to the mass distribution over the planet (**Obliquity**).
- As the earth revolves around the sun, the axis is tilted away from the Sun at the (December solstice) and toward the Sun at the (June solstice), which is the cause of Seasonal change.
- At the equinoxes, the daylight and night are spread evenly, and the equator receives the sun rays directly.
- Notice that the sun rays directed to the equator is changing over time, which is important for Geostationary satellites

- **Equinoxes:** are two periods of time where the equatorial plane of Earth will be aligned with the direction of the sun (i.e. inclination angle = 0)
- An equinox splits Earth's day almost in half, giving us about 12 hours of daylight and 12 of night.
- The inclination angle of the Earth's equatorial plane with respect to the direction of the sun is the angle between the line (joining the centre of the Earth and the sun) and the Earth's equatorial plane
- This angle is not fixed but follows a sinusoidal variation and completes one cycle of sinusoidal variation over a period of 365 days

$$\text{Inclination angle (in degrees)} = 23.4 \ \sin\left(\frac{2\pi t}{T}\right)$$

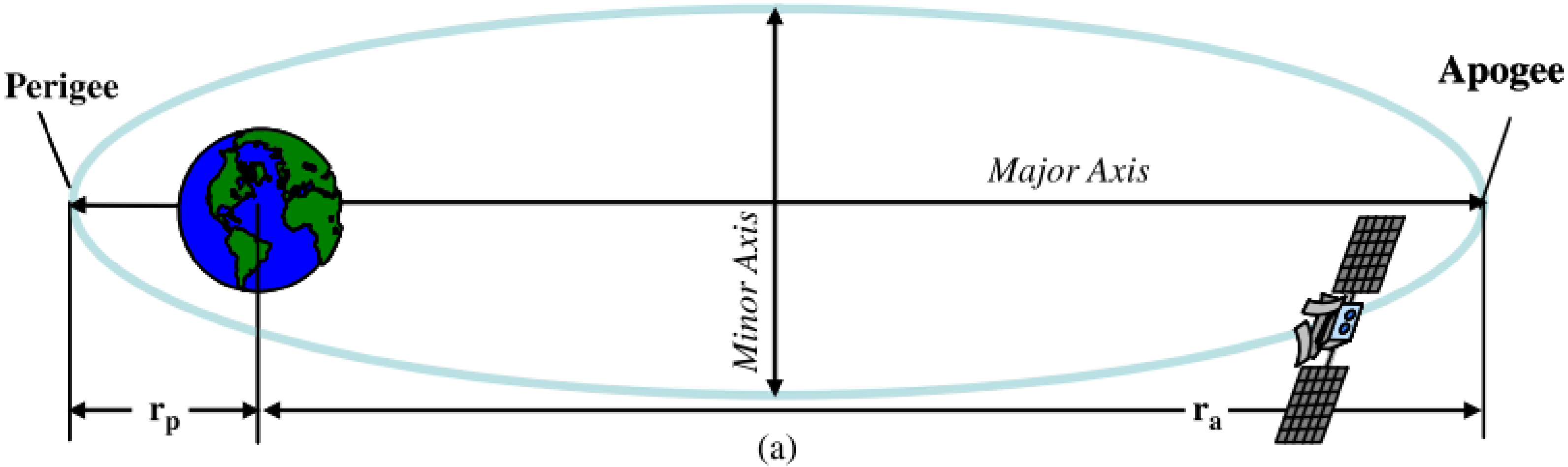
where T = 365 days.



- This angle is zero for $t = T/2$, i.e., on 20-21 March, called the **spring equinox**, and 22-23 September, called the **autumn equinox**.

- Solstices: are the times when the inclination angle is at its maximum, i.e. 23.4◦ (called the summer solstice, and the winter solstice).

Eccentricity in terms of Apogee and Perigee



r_a = the distance from the center of the earth to the apogee point

r_p = the distance from the center of the earth to the perigee point

$$e = \frac{r_a - r_p}{r_a + r_p} = \frac{\text{apogee} - \text{perigee}}{\text{apogee} + \text{perigee}} = \frac{\text{apogee} - \text{perigee}}{2a}$$

- Some references define Apogee and Perigee heights above the earth surface by subtracting the earth radius from r_a and r_b

$$h_a = r_a - R \qquad h_p = r_p - R$$

Eccentricity in terms of Apogee and Perigee

- ✓ **Using the following relation,**

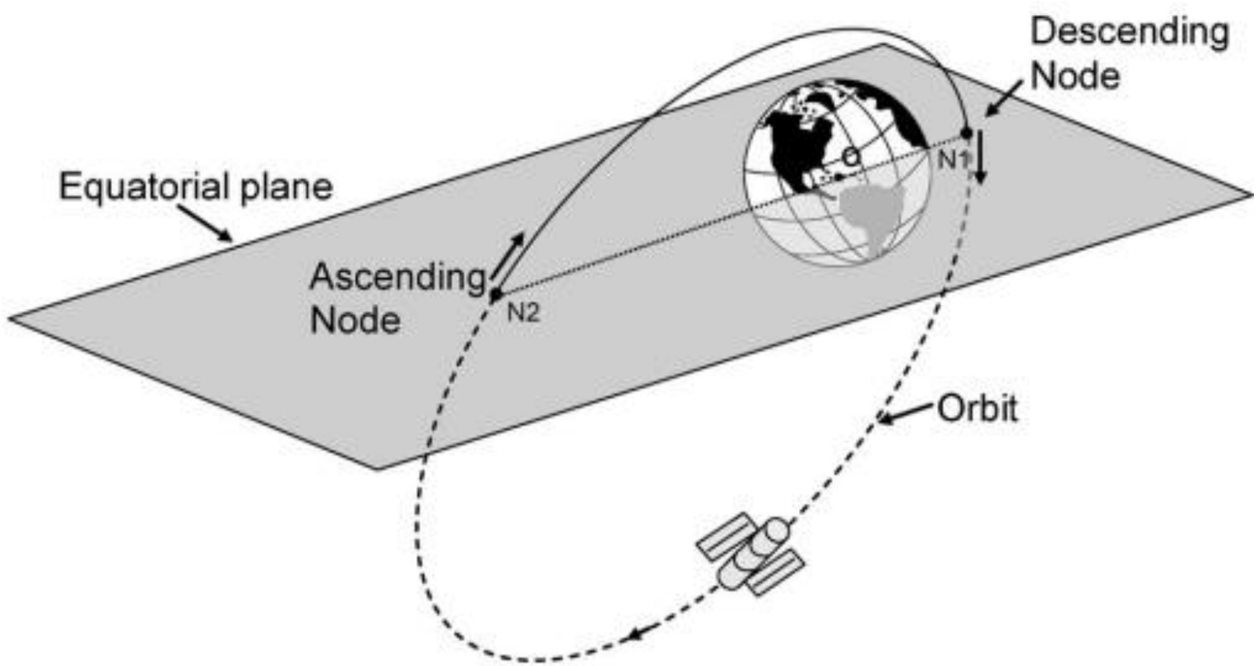
$$e = \frac{r_a - r_p}{r_a + r_p}$$

Prove the following .

$$r_a = a(1 + e)$$

$$r_p = a(1 - e)$$

Ascending and Descending Nodes



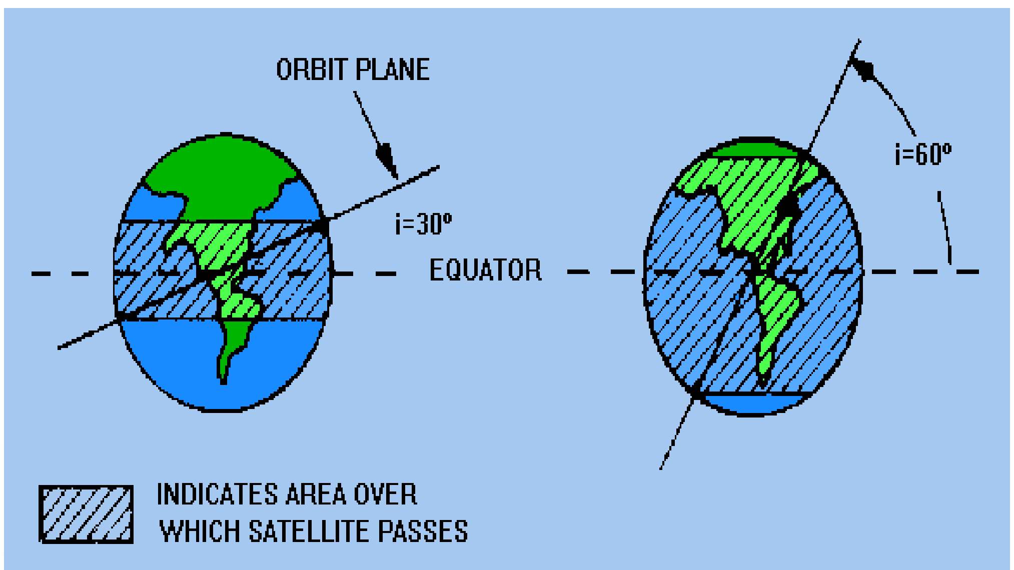
- Ascending Node : the point where the orbit crosses the equatorial plane, going from south to north.
- Descending Node: the point where the orbit crosses the equatorial plane, going from north to south.
- **Line of Nodes** – the line joining the ascending and descending nodes through the center of the earth.

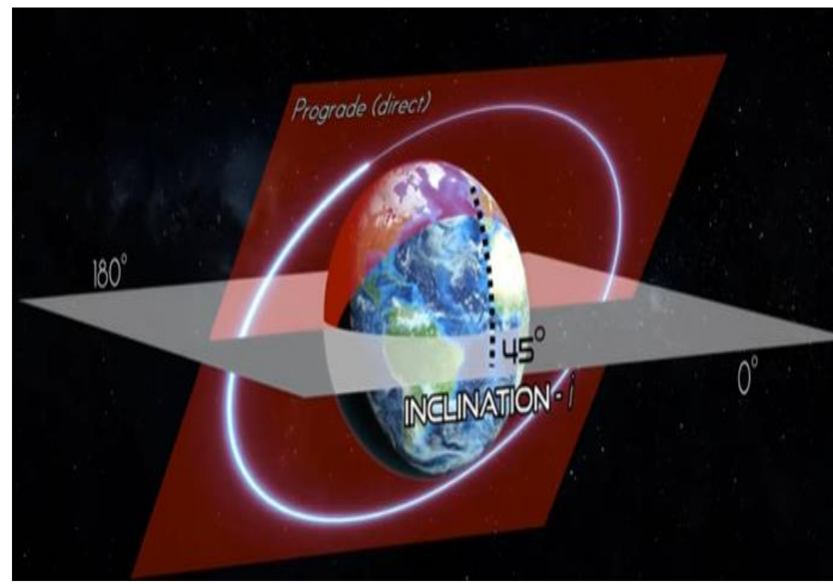
Inclination

- Inclination (i) is used to describe the tilt of the satellite orbit
- Inclination is the angle that the orbital plane of the satellite makes with the Earth's equatorial plane.

$$180^{o} > i > 0^{o}$$

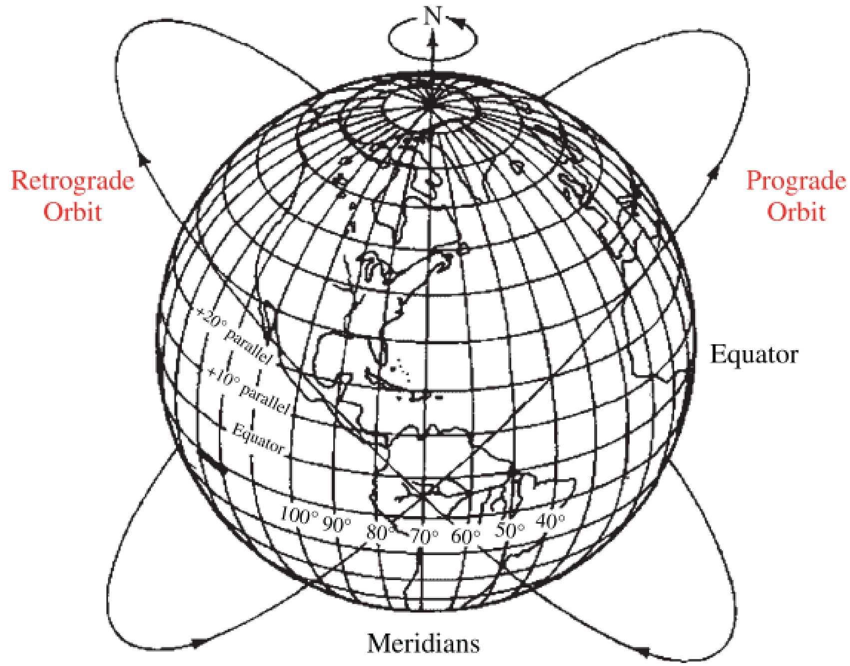
- ✓ A satellite rotating in the equatorial plane have i = 0 (Equatorial Orbit)
- ✓ A satellite that has an inclination angle of 90 is in a polar orbit.
- ✓ A satellite that is in an orbit with some inclination angle is in an inclined orbit.





- **Notice that inclination affects the areas where the satellite passes**

Inclination for Prograde and retrograde orbits



Retrograde or indirect orbit

$180^{o} > i > 90^{o}$

Prograde or direct orbit

$90^{o} > i > 0^{o}$

- ✓ The satellite rotates opposite to the direction of the earth rotation
- ✓ The satellite rotates in the direction of the earth rotation (east/counter-clockwise)

➢ Most satellites are launched in a prograde orbit

Inclination for Prograde and retrograde orbits

- Retrograde orbit :

  - If the satellite is orbiting in the opposite direction as Earth's rotation or in the same direction with an angular velocity less than that of Earth ($\omega_s < \omega_e$)

- Posigrade orbit or Prograde:

  - If the satellite is orbiting in the same direction as Earth's rotation (counterclockwise) and at an angular velocity greater than that of Earth ($\omega_s > \omega_e$)

✓ Both cases are considered a **nonsynchronous orbit satellites**, where the position of satellites are continuously changing in respect to a fixed position on Earth.

References

https://www.youtube.com/watch?v=tX3Y5bzNDiU

Thank you